

Wave Motion

Week 13, Lesson 2

- **Wave Propagation, Wave Types**
- **Wave Terminology**
- **Speed of a Transverse Wave**
- **Standing Waves**
- **Resonance**

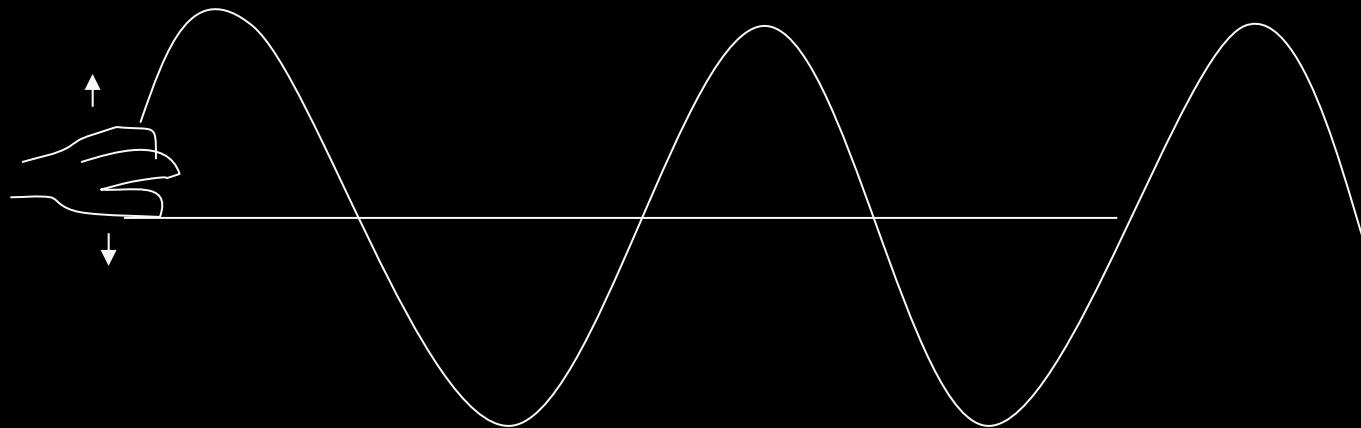
References/Reading Preparation:

Schaum's Outline Ch. 22

Principles of Physics by Beuche – Ch.14

Wave Motion

A wave has been generated on a string by sinusoidal vibration of the hand at its end, as shown.



The wave that is now on the string furnishes a record of earlier vibrations at the source. Energy is carried by the wave from the source (the hand) to the right, along the string.

This direction, the direction of energy transport, is called the *direction of propagation* of the wave.

Each particle of the string vibrates up and down, perpendicular to the line of propagation.

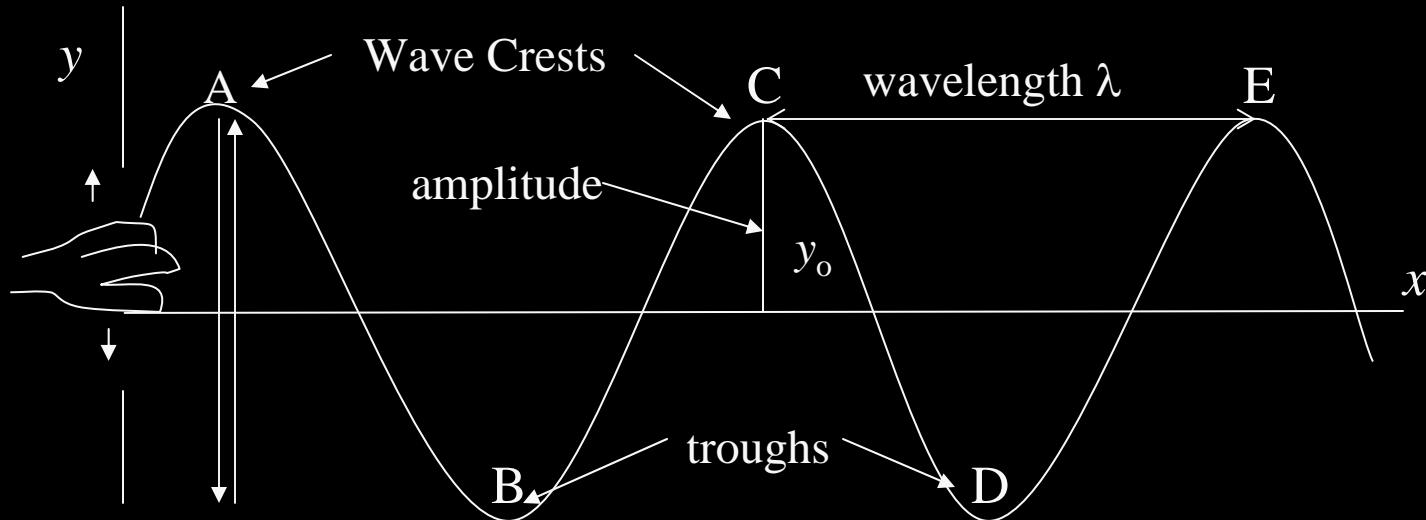
Any wave in which the vibration direction is perpendicular to the direction of propagation is called a *transverse wave*.

Typical transverse waves besides those on a string include electromagnetic waves – light and radio waves.

Sound waves are *longitudinal* (or *compressional*) waves since the vibration direction is parallel to the direction of propagation.

Wave Terminology

The *period of vibration* (T) is the time taken for a particle such as the one at A to move through one complete cycle – down from point A and then back to A.



The *frequency of vibration* (f) is the number of such vibrations executed by the particle each second. $f = 1/T$ unit is the $\text{Hz} = \text{1s}^{-1}$

As time goes on, the crests and troughs move to the right with speed, v .

The wavelength λ is the distance along the propagation between corresponding points on the wave.

In time T (the period), a crest moving with average speed v will move a distance λ (one wavelength) to the right.

Using the formula for distance traveled from linear motion:

$$s = \bar{v}t$$

We get:

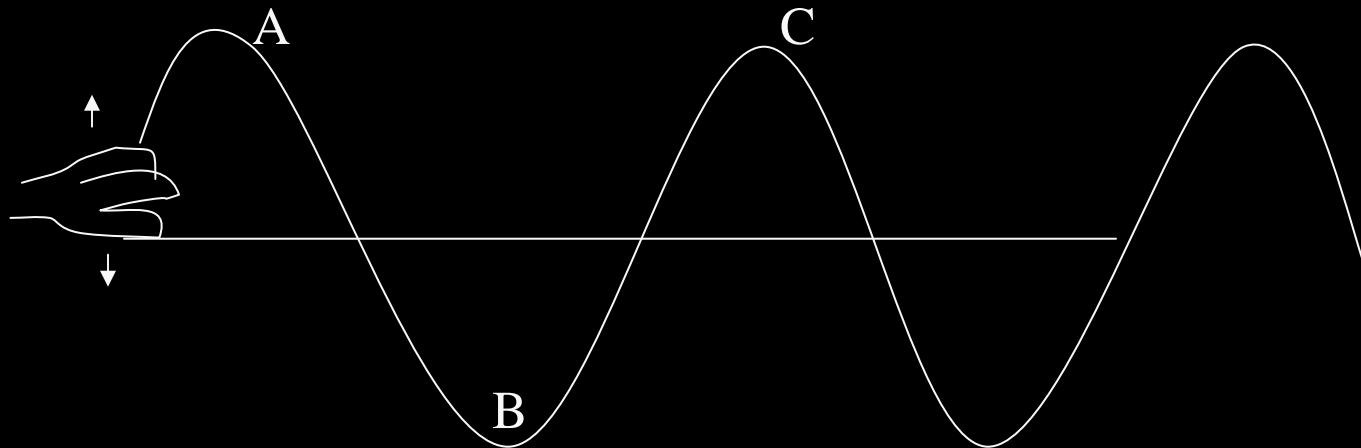
$$\lambda = vT = v/f$$

This relation holds for all waves – not just waves on a string.

In-Phase Vibrations

In-phase vibrations exist at two vibration points of a wave if the points undergo vibration in the same direction together.

For example, the particles of the string at points A and C vibrate in phase, since they move up and down together.



Vibrations are in phase if the points are a whole number of wave lengths apart.

The pieces of the string at A and B vibrate opposite to each other. These vibrations are said to be 180° , or half a cycle, *out of phase*.

Speed of a Transverse Wave

The speed of a transverse wave on a stretched string or wire is:

$$v = \sqrt{\text{tension in string/mass per unit length of string}}$$

Standing Waves

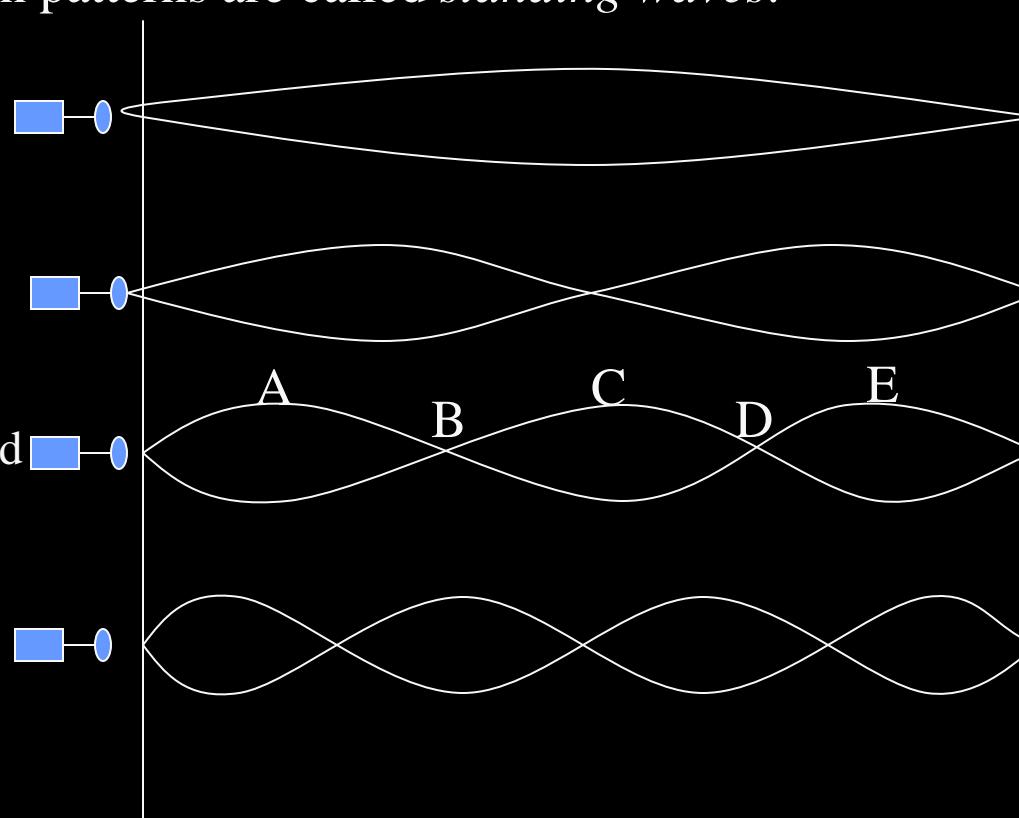
At certain vibration frequencies, a string will *resonate* – that is, it will vibrate with large amplitude in vibration patterns similar to those shown below and will appear as a blur.

These and similar vibration patterns are called *standing waves*.

The stationary points, B,D, are called *nodes*.

The points of greatest motion, A, C, E, are called *antinodes*.

The distance between adjacent nodes (or antinodes) is $\frac{1}{2} \lambda$, and is called a *segment*.



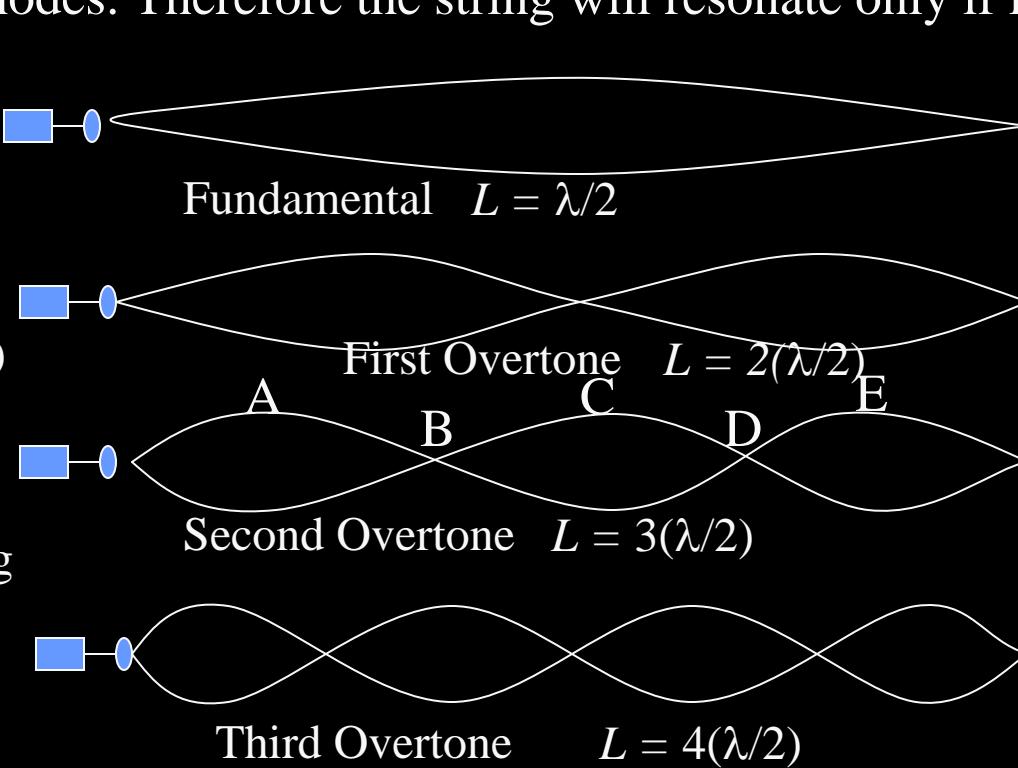
Conditions for Resonance

Looking at our figure, we see that the string always resonates in whole segments – where a segment is the distance between adjacent nodes or antinodes.

The fixed ends are always nodes. Therefore the string will resonate only if it is one segment long, or two segments long, or so on.

Since the length of a segment is $\frac{1}{2} \lambda$, the string can resonate only if it is $\lambda/2$ long, or $2(\lambda/2)$ or $3(\lambda/2)$ long, and so on.

L = length of resonating string



In general, for resonance of a string fastened at both ends,

$$L = n\lambda_n/2$$

Where $n = 1, 2, 3, \dots$

And L is the Length of the resonating string

And λ_n is the wavelength when the string resonates in n segments.

Since wavelength is related to frequency by $\lambda = v/f$, it can be seen that a string of fixed length resonates only to certain special frequencies.

These discrete values of resonance frequency are integer multiples of the fundamental resonance frequency f_1 .

$$f_n = v/\lambda_n = \frac{v}{2L/n} = nf_1$$

Harmonics

Often the resonance frequencies of a taut string are connected with the production of music.

The fundamental frequency f_1 is sometimes referred to as the *first harmonic*, with f_2, f_3, f_4 , and f_n as the second, third, forth, and n th harmonics.

Thus the term *harmonic* in general refers to a vibration of a *single sinusoidal wave frequency*, and the term *simple harmonic motion* refers to periodic motion that can be described by a sine or cosine function with a single frequency.

Harmonics example

The speed of a wave on a particular string is 24 m/s. If the string is 6.0 m long, to what driving frequencies will it resonate?

Answer:

For resonance to occur, the length of the string must be an integral number of half wavelengths. Or, $\lambda_n = 2L/n$

Therefore, $\lambda_1 = 12\text{m}/1 = 12.0\text{ m}$; $\lambda_2 = 12\text{ m}/2 = 6.0\text{ m}$; $\lambda_3 = 12\text{m}/3 = 4.0\text{ m}$

Now, $f_n = v/\lambda_n$

Therefore, $f_1 = 24\text{ m/s} / 12\text{ m} = 2.0\text{ Hz}$

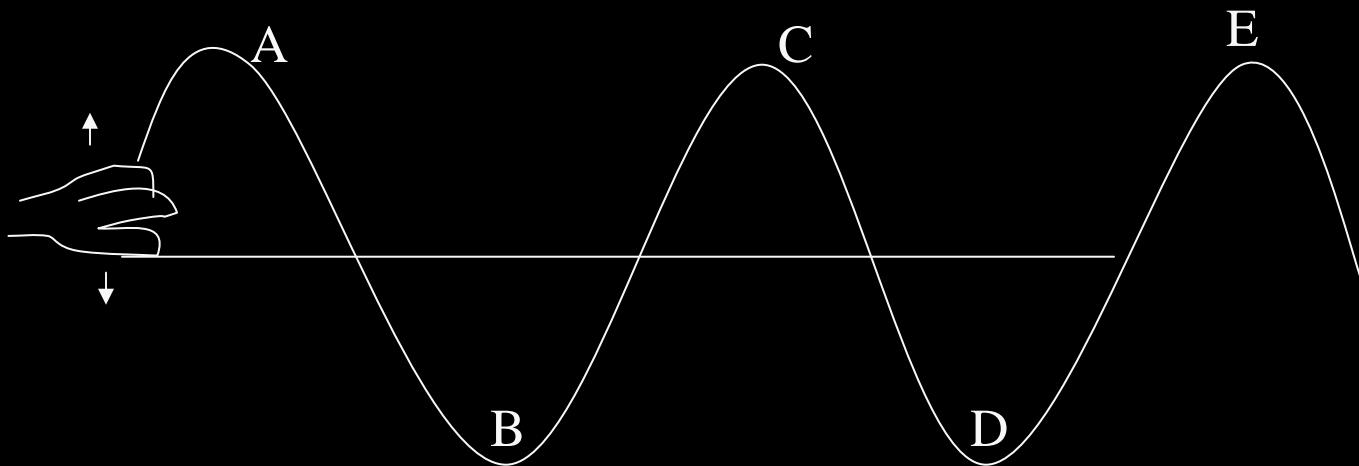
$$f_2 = 24\text{ m/s} / 6.0\text{ m} = 4.0\text{ Hz}$$

$$f_3 = 24\text{ m/s} / 4.0\text{ m} = 6.0\text{ Hz}$$

Worked example

Suppose that the following figure represents a 50 Hz wave on a string. Take Distance y_0 to be 3 mm, and distance AE to be 40 cm. What is:

- a) The amplitude,
- b) The wavelength, and
- c) The speed of the wave.



(ans. 3.0 mm, 20 cm, 10 m/s)

Worked example

A horizontal cord 5 m long has a mass of 1.45 g. What must be the tension in the cord if the wavelength of a 129 Hz on it is to be 60 cm? How large a mass must be hung from its end (say, over a pulley) to give it this tension?

(ans. 1.50 N, 0.153 kg)

Worked example

A string vibrates in five segments to a frequency of 460 Hz.

- a) What is its fundamental frequency?
- b) What will cause it to vibrate in three segments?

Longitudinal Waves

Longitudinal (Compressional Waves) occur as lengthwise vibrations of air columns, solid bars, and the like.

At resonance, nodes exist at fixed points, such as the closed end of an air column in a tube, or the location of a clamp on a bar.

The resonance diagrams shown earlier are used to display the resonance of longitudinal waves as well as transverse waves. However, for longitudinal waves, the diagrams are mainly schematic and are used simply to indicate the locations of nodes and antinodes.

In analyzing these diagrams, we use the fact that the distance between node and adjacent antinode is $\frac{1}{4} \lambda$.

Worked example

Compressional (sound) waves are sent down an air-filled tube 90 cm long and closed at one end. The tube resonates at several frequencies, the lowest of which is 95 Hz. Find the speed of sound waves in air.

(ans. 342 m/s)